Mark Scheme (Results)
Summer 2008
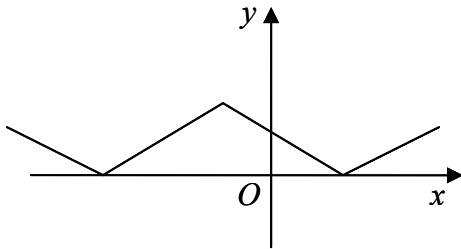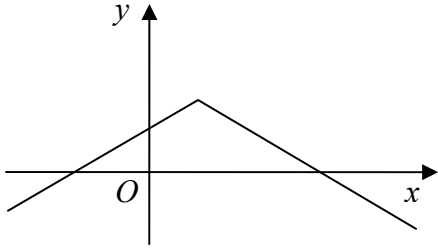
GCE Mathematics (6665/01)

June 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$$e^{2x+1} = 2$$ $$2x+1 = \ln 2$$	M1
	$$x = \frac{1}{2}(\ln 2 - 1)$$	A1 (2)
(b)	$$\frac{dy}{dx} = 8\,e^{2x+1}$$	B1
	$$x = \frac{1}{2}(\ln 2 - 1) \quad \Rightarrow \quad \frac{dy}{dx} = 16$$	B1
	$$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$$	M1
	$$y = 16x + 16 - 8\ln 2$$	A1 (4)
		[6]

Question Number	Scheme	Marks
2. (a)	$$R^2 = 5^2 + 12^2$$ $$R = 13$$ $$\tan\alpha = \frac{12}{5}$$ $$\alpha \approx 1.176 \qquad\qquad\qquad\qquad \text{cao}$$	M1 A1 M1 A1 (4)
(b)	$$\cos(x-\alpha) = \frac{6}{13}$$ $$x - \alpha = \arccos\frac{6}{13} = 1.091\ \ldots$$ $$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ \ldots \qquad \text{awrt 2.3}$$ $$x - \alpha = -1.091\ \ldots \qquad\qquad \text{accept} \ldots = 5.19\ \ldots \text{ for M}$$ $$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ \ldots \quad \text{awrt 0.084 or 0.085}$$	M1 A1 A1 M1 A1 (5)
(c)(i)	$$R_{\max} = 13 \qquad\qquad\qquad\qquad\qquad \text{ft their } R$$	B1 ft
(ii)	At the maximum, $\cos(x-\alpha) = 1 \ or \ x - \alpha = 0$ $$x = \alpha = 1.176\ \ldots \qquad \text{awrt 1.2, ft their } \alpha$$	M1 A1ft (3) **[12]**

Question Number	Scheme		Marks	
3.	(a)	$\vee\!\!\vee$ shape Vertices correctly placed	B1 B1	(2)
	(b)	$\wedge$ shape Vertex and intersections with axes correctly placed	B1 B1	(2)
	(c)	$P:(-1, 2)$ $Q:(0,1)$ $R:(1, 0)$	B1 B1 B1	(3)
	(d)	$x > -1$; $\quad 2 - x - 1 = \dfrac{1}{2}x$ Leading to $x = \dfrac{2}{3}$ $x < -1$; $\quad 2 + x + 1 = \dfrac{1}{2}x$ Leading to $x = -6$	M1 A1 A1 M1 A1	(5)
				[12]

Question Number	Scheme	Marks
4. (a)	$$x^2 - 2x - 3 = (x-3)(x+1)$$	B1
	$$f(x) = \frac{2(x-1)-(x+1)}{(x-3)(x+1)} \quad \left(or \ \frac{2(x-1)}{(x-3)(x+1)} - \frac{x+1}{(x-3)(x+1)} \right)$$	M1 A1
	$$= \frac{x-3}{(x-3)(x+1)} = \frac{1}{x+1} \quad \boldsymbol{*}$$ cso	A1 (4)
(b)	$$\left(0, \frac{1}{4}\right)$$ Accept $0 < y < \frac{1}{4}, \ 0 < f(x) < \frac{1}{4}$ etc.	B1 B1 (2)
(c) Let $y = f(x)$	$$y = \frac{1}{x+1}$$ $$x = \frac{1}{y+1}$$ $$yx + x = 1$$ $$y = \frac{1-x}{x} \qquad\qquad\qquad\qquad\qquad or \ \frac{1}{x}-1$$	M1 A1
	$$f^{-1}(x) = \frac{1-x}{x}$$	
	Domain of f^{-1} is $\left(0, \frac{1}{4}\right)$ ft their part (b)	B1 ft (3)
(d)	$$fg(x) = \frac{1}{2x^2 - 3 + 1}$$	
	$$\frac{1}{2x^2 - 2} = \frac{1}{8}$$	M1
	$$x^2 = 5$$	A1
	$$x = \pm\sqrt{5}$$ both	A1 (3)
		[12]

Question Number	Scheme		Marks
5.	(a) $$\sin^2\theta + \cos^2\theta = 1$$		
	$\div \sin^2\theta \qquad \dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$		M1
	$\qquad\qquad\qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$ ✱	cso	A1 (2)
	Alternative for (a)		
	$1 + \cot^2\theta = 1 + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$		M1
	$\qquad\qquad\qquad\qquad\qquad = \operatorname{cosec}^2\theta$ ✱	cso	A1
	(b) $\qquad\qquad 2\left(\operatorname{cosec}^2\theta - 1\right) - 9\operatorname{cosec}\theta = 3$		M1
	$2\operatorname{cosec}^2\theta - 9\operatorname{cosec}\theta - 5 = 0 \qquad or \qquad 5\sin^2\theta + 9\sin\theta - 2 = 0$		M1
	$\left(2\operatorname{cosec}\theta + 1\right)\left(\operatorname{cosec}\theta - 5\right) = 0 \quad or \quad \left(5\sin\theta - 1\right)\left(\sin\theta + 2\right) = 0$		M1
	$\operatorname{cosec}\theta = 5 \quad or \quad \sin\theta = \dfrac{1}{5}$		A1
	$\theta = 11.5°, 168.5°$		A1 A1 (6)
			[8]

Question Number	Scheme	Marks
6.	(a)(i) $\dfrac{d}{dx}\left(e^{3x}(\sin x + 2\cos x)\right) = 3e^{3x}(\sin x + 2\cos x) + e^{3x}(\cos x - 2\sin x)$	M1 A1 A1 (3)
	$\left(= e^{3x}(\sin x + 7\cos x)\right)$	
	(ii) $\dfrac{d}{dx}\left(x^3 \ln(5x+2)\right) = 3x^2 \ln(5x+2) + \dfrac{5x^3}{5x+2}$	M1 A1 A1 (3)
	(b) $\dfrac{dy}{dx} = \dfrac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$	M1 $\frac{A1}{A1}$
	$= \dfrac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$	M1
	$= \dfrac{20}{(x+1)^3}$ $\quad*\quad$ cso	A1 (5)
	(c) $\dfrac{d^2 y}{dx^2} = -\dfrac{60}{(x+1)^4} = -\dfrac{15}{4}$	M1
	$(x+1)^4 = 16$	M1
	$x = 1, -3$ $\quad$ both	A1 (3)
		[14]

Note: The simplification in part (b) can be carried out as follows

$$\dfrac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$$

$$= \dfrac{(6x^3+18x^2+18x+6) - (6x^3+18x^2-2x-14)}{(x+1)^4}$$

$$= \dfrac{20x+20}{(x+1)^4} = \dfrac{20(x+1)}{(x+1)^4} = \dfrac{20}{(x+1)^3} \qquad \text{M1 A1}$$

Question Number	Scheme	Marks
7.	(a) $\qquad$ $f(1.4) = -0.568 \ldots < 0$	
	$\qquad$ $f(1.45) = 0.245 \ldots > 0$	M1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.4, 1.45)$	A1 $\qquad$ (2)
	(b) $\qquad$ $3x^3 = 2x + 6$	
	$\qquad$ $x^3 = \dfrac{2x}{3} + 2$	
	$\qquad$ $x^2 = \dfrac{2}{3} + \dfrac{2}{x}$	M1 A1
	$\qquad$ $x = \sqrt{\left(\dfrac{2}{x} + \dfrac{2}{3}\right)}$ $\quad *$ $\qquad$ cso	A1 $\qquad$ (3)
	(c) $\qquad$ $x_1 = 1.4371$	B1
	$\qquad$ $x_2 = 1.4347$	B1
	$\qquad$ $x_3 = 1.4355$	B1 $\qquad$ (3)
	(d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.	M1
	$\qquad$ $f(1.4345) = -0.01 \ldots$	
	$\qquad$ $f(1.4355) = 0.003 \ldots$	M1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.4345, 1.4355)$	
	$\qquad$ $\Rightarrow \alpha = 1.435$, correct to 3 decimal places $\quad *$ cso	A1 $\qquad$ (3)
		[11]
	Note: $\alpha = 1.435\,304\,553 \ldots$	